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Optical absorption in terahertz-driven quantum wells under a magnetic field

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Abstract

We have studied the influence of a terahertz (THz) electric field and a magnetic field on the optical absorption in quantum wells. The studies show that the absorption spectrum becomes much richer in the presence of a THz field and a magnetic field. In a growth-direction polarized THz field, the main features of the quantum-confined Stark effect, such as the red-shift of the absorption edge, the reduction in the magnitude of the main excitonic peaks, and the appearance of additional absorption peaks due to transitions which are forbidden in the absence of a DC field, are retained. In addition, new asymmetric replicas emerge around these peaks. The perpendicular magnetic field leads to quantization of the in-plane motion, and thus enhances the main excitonic peaks and shifts the peaks to the blue, and discretizes the continuum of the absorption spectrum. The magnetic field may leads to the merging or broadening of peaks and replicas.

1. Introduction

The influence of intense terahertz (THz) radiation on the transport and interband transition properties of semiconductor heterostructures has attracted considerable attention recently [1–12]. Many efforts have been made to investigate the intense THz-induced optical absorption and sideband generation in quantum wells (QWs) [5–12]. This is due partially to their potential applications as ultra-high-rate modulators or wavelength division multiplexers in modern long haul optical communications. It was experimentally verified that the presence of a THz field leads to a red-shift of the absorption edge and emission of the optical sideband signals [5–8]. The presence of a strong THz field makes the usual description of mixing of several states within the framework of perturbation theory inadequate. A much more rigorous theoretical approach has been developed recently [9–12]. The theoretical approach was based on the multiband polarization semiconductor Bloch equations (SBE) including both Coulomb and THz field-induced intersubband couplings.

The introduction of a magnetic field generates new features in the electronic and optical properties of semiconductor heterostructures [13–18]. Magnetic fields restrict the free motion of electrons perpendicular to the field and lead to the formation of Landau levels (LLs). The linear and nonlinear optical properties of magnetoexcitons either in bulk semiconductors [19, 20] or in QWs [14, 21–23] have been extensively studied.

In this paper we discussed the linear optical absorption of magnetoexcitons in the presence of intense THz fields along the growth direction by a coherent-wave approach, or equation of motion method [24]. A similar situation has been studied by Hughes and Citrin [25], where the THz field was polarized in the QW plane, and thus is the dynamic Franz–Keldysh effect. On the other hand, our case is the extension of the dynamic quantum-confined Stark effect (QCSE) to magnetoexcitons. The main idea of the coherent-wave approach is to solve directly the Schrödinger equation which describes the electron–hole dynamics in the presence of a Coulomb interaction, quantum confinements, external electric fields and a magnetic field in real space. This approach was used to discuss optical absorption of finite thickness QWs under magnetic fields [26] and DC bias, and proved to be effective and robust [27]. It was also extended to study transient dynamics 40 GHz AC bias [27]. Recently, this approach was used to simulate optical absorption in QWs in the presence of a growth-direction polarized THz field [28]. This paper generalizes the approach to magnetoexcitons.

2. Coherent-wave approach of optical absorption in QWs

Making use of the coherent-wave approach to analysis the exciton-dominated optical absorption, the first step is separating the exciton Hamiltonian into the centre of mass (CM) part and the relative motion (RM) part. The separating process for the case without THz field has be presented in a few review papers or monographs [26, 29, 30]. We consider a GaAs QW with infinite barriers at $z = \pm L/2$ with the z axis along the growth direction ([001] direction), and L the QW width. In the approximation with neglecting light-hole and considering only heavy-hole magnetoexcitons, the exciton Hamiltonian $\hat{H}$ with a magnetic field $\mathbf{B}$ and an electric field $\mathbf{F}$ is

$$\hat{H} = \hat{H}_{\rm e} + \hat{H}_{\rm h} + \hat{H}_{\rm Coul}, \tag{1}$$

where $\hat{H}_{\rm e} = \frac{1}{2m_{\rm e}}(-{\rm i}\hbar\boldsymbol{\nabla}_{\mathbf{r}{\rm e}} + e\mathbf{A}_{\rm e})^2 + e\mathbf{F}\cdot\mathbf{r}_{\rm e}$ and $\hat{H}_{\rm h} = \frac{1}{2m_{\rm h}}(-{\rm i}\hbar\boldsymbol{\nabla}_{\mathbf{r}{\rm h}} - e\mathbf{A}_{\rm h})^2 - e\mathbf{F}\cdot\mathbf{r}_{\rm h}$ are the single-particle Hamiltonians for electrons and holes, respectively, and the Coulomb interaction $\hat{H}_{\rm Coul} = -\frac{e^2}{4\pi\varepsilon|\mathbf{r}_{\rm e}-\mathbf{r}_{\rm h}|}$ correlates the motion of the electron and hole. Here $m_{\rm e}$, $\mathbf{r}_{\rm e}$ and $\mathbf{A}(\mathbf{r}_{\rm e}) = \frac{1}{2}\mathbf{B}\times\mathbf{r}_{\rm e}$ are the mass, coordinate, and vector potential of the electron respectively, and $m_{\rm h}$, $\mathbf{r}_{\rm h}$, and $\mathbf{A}(\mathbf{r}_{\rm h}) = \frac{1}{2}\mathbf{B}\times\mathbf{r}_{\rm h}$ are the mass, coordinate, and vector potential of the hole respectively; e is the elementary charge, and ε is the dielectric constant. It is well known that the magnetoexciton Hamiltonian can be partially separated into the CM part and the RM part by a unitary transformation defined by the operator $U = \exp(-{\rm i}e\mathbf{r}\times\mathbf{B}/2\hbar)$, where $\mathbf{r} = \mathbf{r}_{\rm e} - \mathbf{r}_{\rm h}$ is the relative coordinate of the electron and hole [31]. The transformed Hamiltonian is

$$\hat{H} = \frac{\hat{\mathbf{P}}^2}{2M} + \hat{H}_{\rm RM} + \hat{H}_{\rm int}. \tag{2}$$

Here, $M = m_{\rm e} + m_{\rm h}$ is the total exciton mass, $\hat{\mathbf{P}} = -{\rm i}\hbar\boldsymbol{\nabla}_{\mathbf{R}} - e\mathbf{A}(\mathbf{r})$ is the conserved magnetic CM momentum, $\mathbf{R} = (m_{\rm e}\mathbf{r}_{\rm e} + m_{\rm h}\mathbf{r}_{\rm h})/M$ is the CM coordinate, the Hamiltonian $\hat{H}_{\rm RM}$ describes an RM quasiparticle in the presence of one-body potentials due to the Coulomb interaction and the electric and magnetic field,

$$\hat{H}_{\rm RM} = \frac{\hat{\mathbf{p}}^2}{2m_{\rm r}} - \frac{e^2}{4\pi\varepsilon r} + \frac{e^2}{8m_{\rm r}}(\mathbf{B}\times\mathbf{r})^2 + \frac{e}{2m_{\rm r}}\frac{m_{\rm h}-m_{\rm e}}{M}\mathbf{B}\cdot\hat{\mathbf{L}} + e\mathbf{F}\cdot\mathbf{r}, \tag{3}$$

where m_r is the reduced electron–hole mass, $\hat{\mathbf{p}}$ is the relative momentum operator, and $\hat{\mathbf{L}}$ is the angular momentum operator, while

$$\hat{H}_{\text{int}} = \frac{e}{M}(\hat{\mathbf{P}} \times \mathbf{B}) \cdot \mathbf{r}, \tag{4}$$

describes a two-body interaction between the CM and RM degrees of freedom.

We consider the case where the electric field and magnetic field are both along the growth direction, i.e. $\mathbf{B} = (0, 0, B)$ and the electric field $\mathbf{F} = (0, 0, F)$. The electric field contains DC and AC components,

$$F = F_{\text{DC}} + F_{\text{AC}} \cos(\Omega t), \tag{5}$$

where F_{DC} and F_{AC} are the amplitudes of the DC and AC field respectively, and the frequency Ω lies in the THz range. Since only the relative coordinate wavefunction influences the absorption spectrum of semiconductors and the term of the Zeeman effect vanishes for s-like exciton states seen in one-photon transitions, the RM Hamiltonian can be reduced further to the effective Hamiltonian,

$$\begin{aligned}\hat{H}_{\text{eff}} = &-\frac{\hbar^2}{2m_{\text{r}\parallel}}\nabla_\rho^2 - \frac{\hbar^2}{2m_{\text{e}\perp}}\frac{\partial^2}{\partial z_{\text{e}}^2} - \frac{\hbar^2}{2m_{\text{h}\perp}}\frac{\partial^2}{\partial z_{\text{h}}^2} - \frac{e^2}{4\pi\varepsilon\sqrt{\rho^2 + (z_{\text{e}} - z_{\text{h}})}} \\ &+ \tfrac{1}{8}m_{\text{r}\parallel}\omega_{\text{c}}^2\rho^2 - e[F_{\text{DC}} + F_{\text{AC}}\cos(\Omega t)](z_{\text{h}} - z_{\text{e}}),\end{aligned} \tag{6}$$

where $\omega_c = eB/m_r$ is the cyclotron frequency of the applied magnetic field.

The optical properties of the QW are characterized by the linear optical susceptibility $\chi(t; t')$ which relates the optical field $E(t)$ with the polarization $P(t)$ induced by $E(t)$,

$$P(t) = \int_{-\infty}^{+\infty} \text{d}t'\, \chi(t; t')E(t'). \tag{7}$$

The optical field is assumed to be incident normally on the QW; thus, it is polarized in the QW plane and $E(t) = E_0 \exp(-t^2/\tau^2)$ is the slowly varying envelope. Unlike the case in the absence of a THz field where $\chi(t; t') = \chi(t - t')$, the susceptibility in this case is periodic: $\chi(t + T; t' + T) = \chi(t; t')$, with $T = 2\pi/\Omega$ the THz period. Since we are interested in the response to the continuous-wave (CW) optical fields, we perform a Fourier transform to equation (7) and get

$$\tilde{P}(\omega) = \sum_{-\infty}^{+\infty} \tilde{\chi}_n(\omega; \omega - n\Omega)\tilde{E}(\omega - n\Omega). \tag{8}$$

Here and later on, the tilde sign over quantities indicates that these quantities are defined in the frequency-domain to distinguish them from their time-domain counterparts. The imaginary part of $\tilde{\chi}_0(\omega; \omega)$ describes the linear optical absorption, and the real part describes the refractive index.

The optical polarization of QWs can be calculated from the time-dependent electron–hole envelope wavefunction $\Psi(\rho, z_e, z_h; t)$ by

$$P(t) = d^* \int_{-L/2}^{L/2} \text{d}z\, \Psi(0, z, z; t), \tag{9}$$

where d^* is the complex conjugate of d with d the projection on the direction of the optical polarization of the interband dipole matrix element. The electron–hole envelope wavefunction depends on three arguments: the electron and hole coordinates z_e, z_h in the growth direction and the in-plane relative coordinate ρ. Since the QW width is much smaller than the optical wavelength in the material, the optical response is considered as local. Using the effective mass theory for the envelope function, the envelope function obeys the inhomogeneous Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} = \hat{H}_{\mathrm{eff}} \Psi - d\delta(z_{\mathrm{e}} - z_{\mathrm{h}})\delta(\boldsymbol{\rho})E(t) - \mathrm{i}\hbar\gamma\Psi, \tag{10}$$

where a phenomenological dephasing constant γ is introduced.

In order not to lose the main advantage of the time-domain approach to the solution of the Schrödinger equation, an effective method was suggested by using several very short optical pulses to extract $\tilde{\chi}_n(\omega; \omega - n\Omega)$ [32]. The method proceeds by setting several very short pulses arriving at well defined phases of the THz field, $E_l(t) = E(t - \frac{lT}{N})$, $l = 0, 1, \ldots, N-1$, where l is the number of optical pulses, and solving the Schrödinger equation for each short pulse $E_l(t)$ to get each induced polarization $P_l(t)$ with Fourier transform $P_l(\omega)$. The required susceptibility in the frequency domain can be calculated as

$$\tilde{\chi}_0(\omega; \omega) = \frac{1}{N} \sum_{l=0}^{N-1} \frac{\tilde{P}_l(\omega)}{\tilde{E}_l(\omega)}. \tag{11}$$

3. Influence of THz field and magnetic field on the optical absorption

We solve equation (10) by the finite difference time-domain method. The detailed procedure can be found in [26, 28], and is not presented here. In all of the calculations below, we take the width of QWs, L, to be equal to the bulk exciton Bohr radius as 15.8 nm, the dephasing constant $\hbar\gamma = 2.5$ meV, the time step 0.1 fs, the spatial grid spacing 1.58 nm, and the FWHM of the optical pulse 15.0 fs. To obtain the susceptibility in the presence of a THz field, we used ten short optical pulses to sample the THz period. The material parameters we used are typical for GaAs: $m_{\mathrm{h}\|} = 0.109\, m_0$, $m_{\mathrm{h}\perp} = 0.408\, m_0$, $m_{\mathrm{e}\|} = m_{\mathrm{e}\perp} = 0.067\, m_0$, with m_0 the free electron mass, and the in-plane reduced e–h mass is $m_{\mathrm{r}} = m_{\mathrm{h}\|} m_{\mathrm{e}\|}/(m_{\mathrm{h}\|} + m_{\mathrm{e}\|})$.

Figure 1 shows the optical absorption spectrum in a QW when the DC field F_{DC} is applied along the growth direction. The optical absorption is dominated by the sharp excitonic peaks corresponding to the bound electron–hole pair between the valence and conduction subbands. In the absence of F_{DC}, there are two absorption excitonic peaks which correspond to the transitions with the same subband index, i.e. e1–h1 and e2–h2. The transitions between subbands with different numbers are forbidden. In the presence of bias, the amplitudes of the main excitonic peaks are reduced and their positions are shifted to the red. There emerge a few new peaks corresponding to transitions forbidden in the absence of F_{DC} (in particular, e1–h2, e1–h3, e2–h1, e2–h3). These effects of the DC field on the optical absorption are well-known as the QCSE [33]. Comparing the numerical results here and the original experimental results of figure 5 in [33], we found that the main features are grasped in the present model. The obvious Fano line shape of the optical absorption spectrum due to the unbound e–h states is included directly. The reduction of the main excitonic peaks is due to the reduction in the Coulomb interaction between the electron and hole as they are pulled apart by the applied DC field. The emergence of new excitonic peaks is because the presence of the DC field changes the single-particle wavefunctions and gives rise to transitions between the valence and conduction subbands with different subband indices.

The optical absorption in a QW in a perpendicular magnetic field is shown in figure 2. The magnetic field permits the redistribution of excitonic oscillator strength. The optical absorption spectrum is also dominated by the sharp excitonic peaks. The magnetic field enhances the main excitonic peaks and shifts them to the blue. However, the continuum of the optical absorption spectrum becomes discrete peaks with Lorentzian line shape. Except for the main peaks, the discrete transition energies are separated from each other by approximately the LL. With the

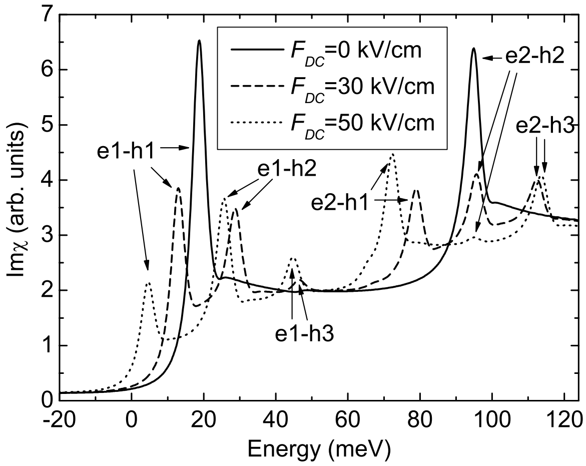

Figure 1. Optical absorption in a QW for $F_{DC} = 0, 30, 50$ kV cm^{-1}. In the absence of a DC field ($F_{DC} = 0$ kV cm^{-1}), the optical absorption is dominated by the main excitonic peaks e1–h1 and e2–h2. In the presence of a DC field ($F_{DC} = 30, 50$ kV cm^{-1}), the main excitonic peaks are shifted to the red and reduced in amplitude, and new excitonic peaks corresponding to e1–h2, e1–h3, e2–h1, e2–h3 which are forbidden in the absence of a DC field appear. The new peaks increase with F_{DC} increasing.

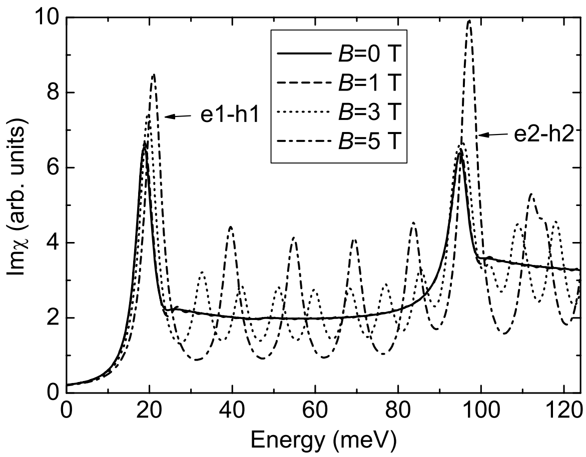

Figure 2. Optical absorption in a QW for $B = 0, 1, 3, 5$ T. In the presence of magnetic field B, the main excitonic peaks e1–h2 and e2–h2 are enhanced and shifted to the blue, and the continuum of absorption become discrete. The modulation and separation increase with B increasing.

increase of the magnetic field the modulation of the continuum absorption and the blue-shift of the main excitonic peaks are more prominent. These effects are because the conduction and valence bands break down into LLs, the in-plane kinetic energy of electron motion is quenched, and the Coulomb interaction effects are enhanced. But when the magnetic field is low, as in the trace of 1 T in figure 2, the influence of the magnetic field is barely discernible. This is due to the dominance of Coulomb interaction over the Landau quantization. The distortion of the first magnetoexciton peak next to the e2–h2 at $B = 5$ T can be noticed; this may be due to the interplay of the corresponding magnetoexciton with the e2–h3 exciton.

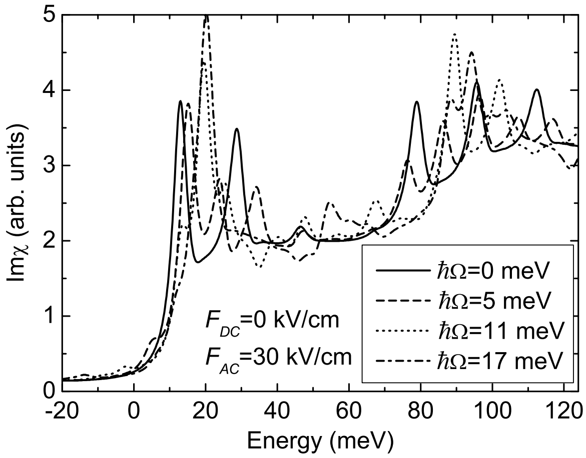

Figure 3. Optical absorption in a QW for a THz field $F_{AC} = 30$ kV cm^{-1} with $\hbar\Omega = 0, 5, 11, 17$ meV. The THz field gives rise to an effect similar to the QCSE in DC field. The red-shift of the main exciton peak and reduction in amplitude reduce with $\hbar\Omega$ increasing. In addition, some asymmetric replicas appear around excitonic peaks.

The linear optical absorption in two-dimensional magnetoexcitons has been solved based on perturbation expansions in terms of Landau orbitals; however, up to 1700 bases were used, because the expansion in terms of Landau orbitals converges very slowly due to the equidistance of LLs [14].

Figure 3 shows the optical absorption in a QW in the presence of a THz field polarized along the growth direction with various frequencies and fixed field strength $F_{AC} = 30$ kV cm^{-1}. The presence of the THz field also gives rise to the red-shift of the main excitonic peaks and reduction of their height as in the DC field case, but these effects are both smaller compared with what one would observe with a DC field of the same strength. With the increase of the frequency, both the red-shift and the reduction of excitonic peaks are weaker. The THz field also leads to the transitions forbidden in the absence of field. In addition, some replicas form around the main excitonic peaks and these new transition peaks in the absorption spectrum; thus the absorption spectrum becomes much richer. The replicas show obvious asymmetric Fano line shape due to the Fano resonance coupling to the continuum part of the absorption. When the frequency of the THz field increases, the replicas move away from the central peak. For high frequency, resonance between the levels can occur and lead to the splitting of the excitonic line.

Figures 4(a) and (b) show the optical absorption in a QW in the presence of a magnetic field and DC field with $F_{DC} = 30$ and 50 kV cm^{-1}, respectively. The red-shift of the excitonic peaks is dominated by the applied DC field, and slightly alleviated by magnetic shift. The amplitudes of the peaks are increased owing to Landau quantization of the in-plane motion. The increase is more prominent for DC field $F_{DC} = 30$ kV cm^{-1} than for $F_{DC} = 50$ kV cm^{-1}. Some peaks, such as e1–h2 for $B = 3$ T and e2–h2 for $B = 5$ T as shown in figure 4(a) and e2–h1 for $B = 5$ T as shown in figure 4(b), become split. It can be seen that the cyclotron resonance peaks around the excitonic peaks e2–h1 and e2–h3 for $B = 3$ T merged into the excitonic peaks and form broadened peaks in figure 4(b). The splitting is due to the resonance of LLs. Comparing figures 4(a) and (b), we found that the line shapes of some magnetoexciton peaks change from symmetric Lorentzian shape to asymmetric Fano shape.

To see the interplays of the DC field, THz field and magnetic field, figure 5 shows the optical absorption in a QW when these fields are present simultaneously. In this case, the

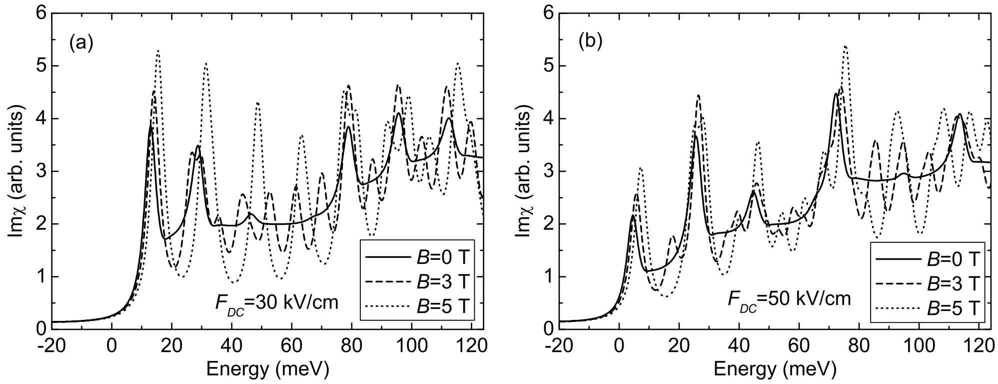

Figure 4. Optical absorption in a biased QW for $B = 0, 3, 5$ T, $F_{\mathrm{DC}} = 30$ kV cm^{-1} (a) and $F_{\mathrm{DC}} = 50$ kV cm^{-1} (b). The red-shift of the excitonic peaks is dominated by the applied DC field and slightly alleviated by the magnetic field. The amplitudes of the peaks are increased by the magnetic field. Some peaks, such as e1–h2 for $B = 3$ T and e2–h2 for $B = 5$ T in (a) and e2–h1 for $B = 5$ T in (b), become split. The cyclotron resonance peaks around the excitonic peaks e2–h1 and e2–h3 at $B = 3$ T merged into the excitonic peaks and form broadened peaks in (b).

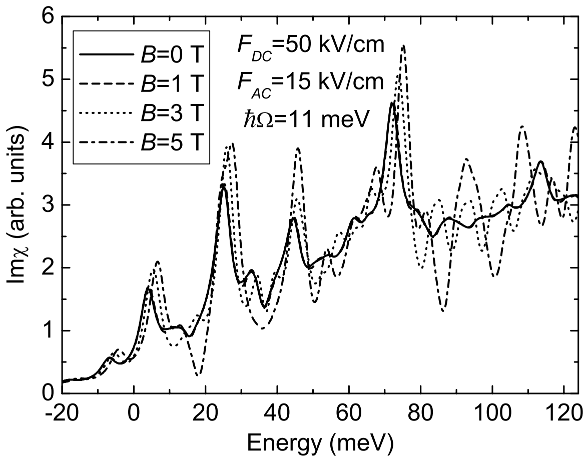

Figure 5. Optical absorption in a QW for $B = 0, 1, 3, 5$ T, $F_{\mathrm{DC}} = 50$ kV cm^{-1} and $F_{\mathrm{AC}} = 15$ kV cm^{-1}, $\hbar\Omega = 11$ meV. The absorption spectrum becomes much richer. The red-shift and reduction of the main excitonic peak due to the DC field and replicas due to the THz field are alleviated slightly by magnetic field. Some replicas merge into broadened excitonic peaks.

red-shifts and the reduction of the peaks due to the electric fields are alleviated by the magnetic field. More importantly, the excitonic peaks and some replicas around these peaks are enhanced significantly. Some peaks become split and some merge into a broadened peak. The replicas show Fano line shape when a THz field is applied. Generally, Fano interference cannot be observed in magneto-optical experiments on two-dimensional quantum well structures since in quantum wells the necessary continuum states have been removed by the layered structure; however, our numerical result shows that when DC and THz fields are applied, the situation is changed. Another reason for the Fano line shape is the finite thickness of our QW. Though the fact that the DC field can lead to Fano resonance has been investigated theoretically and experimentally [34, 35], the Fano resonance is due to the coupling of the exciton or magnetoexciton to the continuum of lower-lying Wannier–Stark states in superlattices and without a THz field. We have shown that the interplays of DC field, THz field and magnetic field can make the optical absorption spectrum much richer in QWs.

4. Conclusion

In conclusion, we have studied the influence of an electric field and a magnetic field on the optical absorption in QWs. The absorption spectrum becomes much richer in a THz field and a magnetic field. The effect of growth-direction DC bias is the well-known QCSE. In the presence of a growth-direction polarized THz field, the main features of QCSE, such as red-shift of the absorption edge, reduction in the magnitude of the main excitonic peaks, and appearance of additional absorption peaks due to transitions which are forbidden in the absence of electric field, are retained. In addition, new asymmetric replicas appear around these peaks. The introduction of a magnetic field enhances the main excitonic peaks and shifts the peaks to the blue. The continuum of the absorption spectrum becomes discrete due to Landau quantization of the in-plane motion. When an electric field (both DC and THz) and a magnetic field are applied simultaneously, the shift of the positions and reduction of the amplitudes of excitonic peaks are dominated by the DC field and alleviated by magnetic field. This study generalizes the recent studies of the dynamical QCSE for the case in the presence of a magnetic field. It shows that the optical absorption in a QW may be controlled effectively by applying a suitable DC field, THz field and magnetic field. This may be used to help design new photoelectronic devices.

Acknowledgments

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